Parallel implementation of $CNOT^N$ and C_2NOT^2 gates via homonuclear and heteronuclear Förster interactions of Rydberg atoms

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Физика Ультрахолодных Атомов – 2022
"Physics of ultracold atoms - 2022"
December 21, 2022.

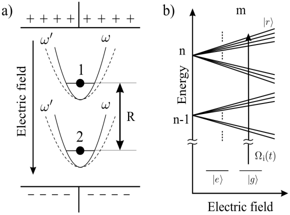
Outline

Manuscript available on arXiv: 2206.12176

Alkali Rydberg atoms

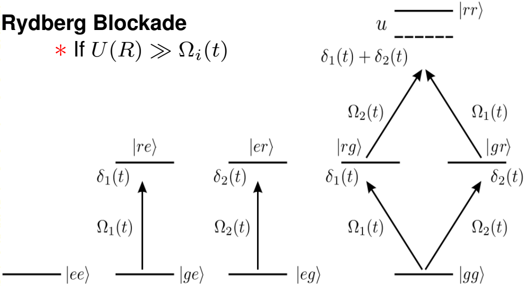
Rydberg atom is a neutral atom with one or more electrons in a highly excited state with principal quantum number $n \gg 1$.

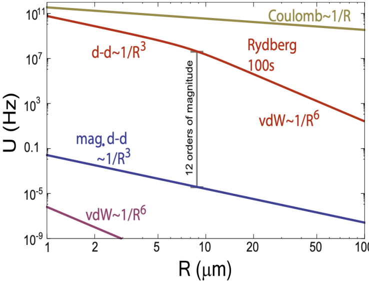
Alkali metals, particularly rubidium and cesium, are the species of atoms most commonly used for Rydberg atom experiments due to their single valence electrons.



Rydberg Blockade

* If $U(R) \gg \Omega_i(t)$





PRL 85, 2208 (2000).

Rev. Mod. Phys, 82, 3 (2313) (2010).

Le Roy radius $\quad R_{\text{LR}} \propto n^3$

$U_{\text{d-d}}(R) = \frac{C_3}{R^3} \quad C_3 \propto n^4$

$U_{\text{vdW}}(R) = \frac{C_6}{R^6} \quad C_6 \propto n^{11}$

ARC, Comp. Phys. Com. 220, 319 (2017).

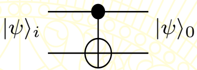
C-NOT gate and Bell states

$$\text{CNOT}|x\rangle|y\rangle = |x\rangle|x \otimes y\rangle$$

$$\text{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

CNOT gate

input	**output**
$\|00\rangle$	$\|00\rangle$
$\|01\rangle$	$\|01\rangle$
$\|10\rangle$	$\|11\rangle$
$\|11\rangle$	$\|10\rangle$



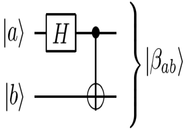
Nielsen and Chuang (2000)
[*Quantum computation and quantum information*]

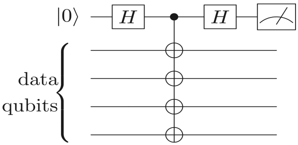
Since superposition of input states leads to a superposition of the corresponding output states

$$|00\rangle \xrightarrow{\text{Hadamard gate}} \frac{1}{\sqrt{2}}\left(|00\rangle + |10\rangle\right) \xrightarrow{\text{C-NOT gate}} \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

Bell states

$$|\Phi^{\pm}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle \pm |11\rangle\right) \qquad |\Psi^{\pm}\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle \pm |10\rangle\right)$$





PRA 96, 052320 (2017)

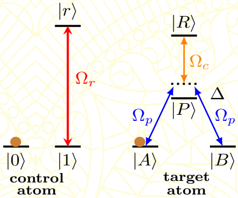
Hadamard gate

$$H|0\rangle = |+\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

$$H|1\rangle = |-\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)$$

Parallel Implementation of C-NOT^N gates

(I)Blocking gate: $|0\rangle|A^N\rangle \rightarrow |0\rangle|A^N\rangle$
$|0\rangle|B^N\rangle \rightarrow |0\rangle|B^N\rangle$.



Blocking scheme

Parallel Implementation of C-NOT^N gates

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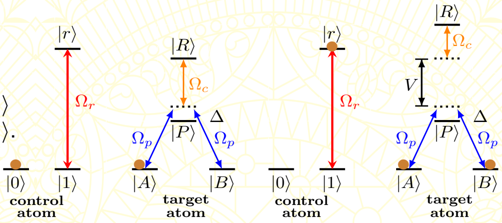
(II)Transfer gate: $$\begin{aligned}|1\rangle|A^N\rangle &\to -(-1)^N|1\rangle|B^N\rangle\\ |1\rangle|B^N\rangle &\to -(-1)^N|1\rangle|A^N\rangle.\end{aligned}$$



Blocking scheme **Transfer scheme**

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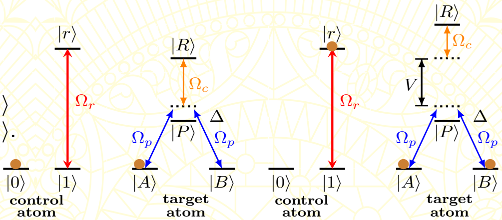
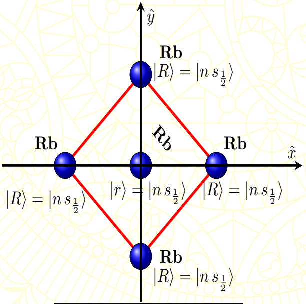
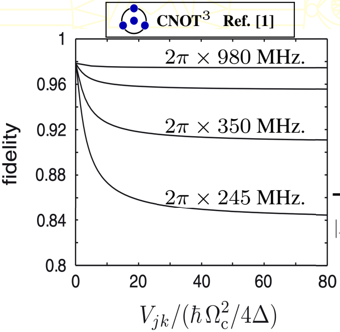


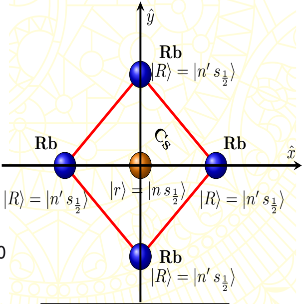
Blocking scheme **Transfer scheme**



Homonuclear architecture

— **Refs: [1-4]**





Heteronuclear architecture

—**[This study]**

[1] PRL102, 170502 (2009),
[2] Nature Physics 6, 382 (2010),
[3] New J. Phys. 16 053045 (2014),
[4] PRA 96, 052320 (2017).

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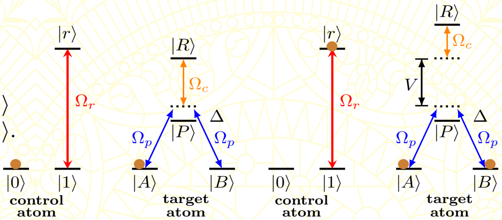
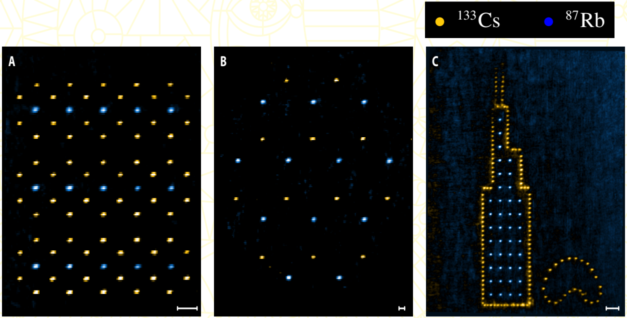
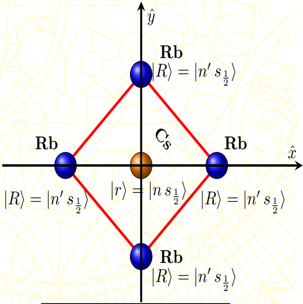


Blocking scheme **Transfer scheme**



Heteronuclear architecture

- PRX 12, 011040 (2022)



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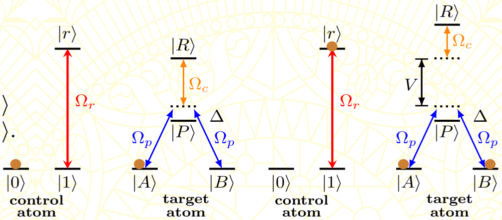
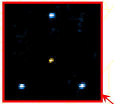
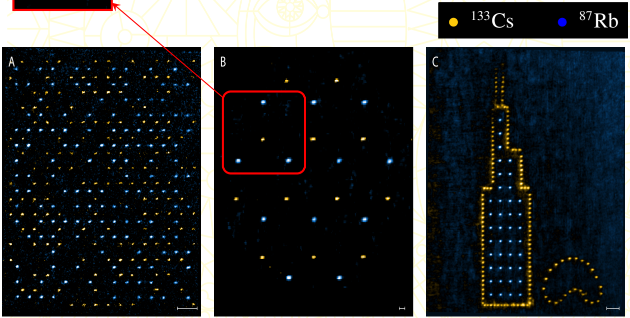


Blocking scheme **Transfer scheme**

CNOT^3 **CNOT^4** **C$_2$NOT^2**





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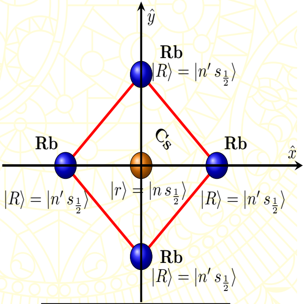

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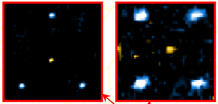
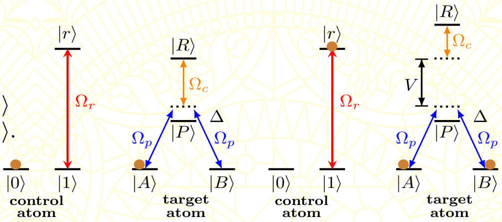
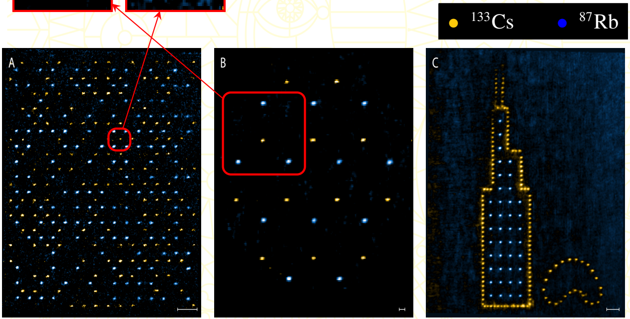
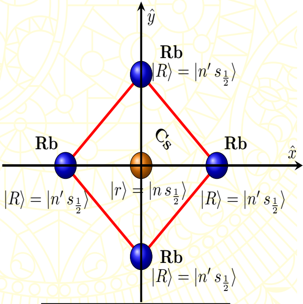
CNOT^3 **CNOT^4** **C$_2$NOT^2**





Blocking scheme **Transfer scheme**





Heteronuclear architecture

- **PRX 12, 011040 (2022)**

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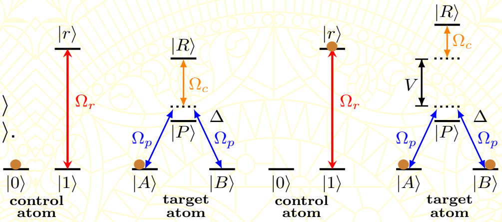

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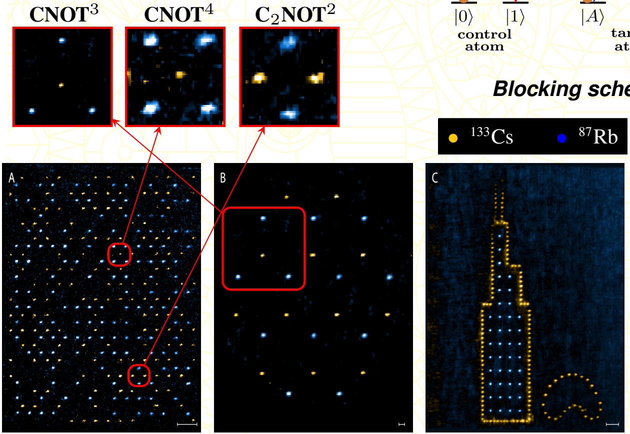

Heteronuclear architecture

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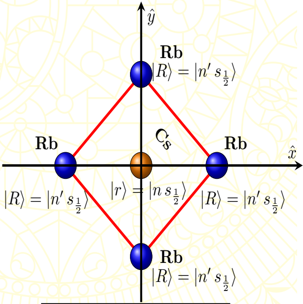

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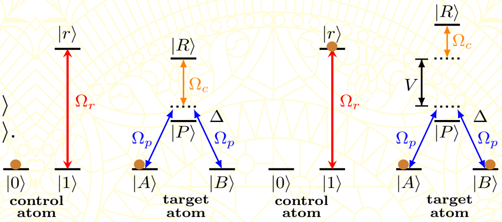
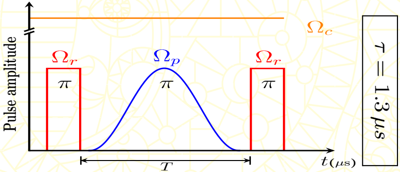
PRL102, 170502 (2009)



Blocking scheme **Transfer scheme**

The Hamiltonian of control atom can be written as:

$$\hat{H}_{\rm C} = \frac{\hbar}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \Omega_r \\ 0 & \Omega_r & 0 \end{pmatrix}.$$



The Hamiltonian of target atom can be written as:

$$\hat{H}_{\rm T} = \frac{\hbar}{2}\begin{pmatrix} 0 & 0 & \Omega_p(t) & 0 \\ 0 & 0 & \Omega_p(t) & 0 \\ \Omega_p(t) & \Omega_p(t) & -2\Delta & \Omega_c \\ 0 & 0 & \Omega_c & 0 \end{pmatrix}.$$

$$\Omega_p(t) = \sqrt{\frac{16\pi\Delta}{3T}}\sin^2\left(\frac{\pi t}{T}\right)$$

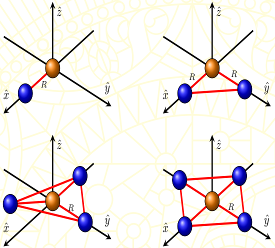
$$\int_0^{\rm T} \Omega_p^2(t)\,dt = 2\pi\Delta$$

The Hamiltonian of the combined system for N target atom

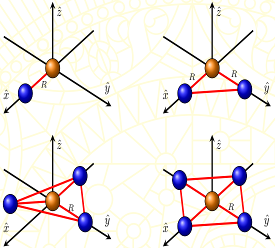
$$\hat{H} = \hat{H}_C \otimes \hat{I} + \hat{I} \otimes \hat{\mathcal{H}}_T + \sum_j^N V_{ct_j}|r\rangle\langle r| \otimes |R\rangle_j\langle R| + \sum_{j\neq k}^N V_{t_j t_k}|R\rangle_j\langle R| \otimes |R\rangle_k\langle R|.$$

control-target interaction target-target interaction

Spatial configurations of CNOT^N gates



Spatial configurations of CNOT^N gates



$$\hat{\mathcal{H}}_t = \hat{H}_{t_1} \otimes \hat{I}_{\mathcal{N}_t} \otimes \hat{I}_{\mathcal{N}_t} \otimes \hat{I}_{\mathcal{N}_t} + \hat{I}_{\mathcal{N}_t} \otimes \hat{H}_{t_2} \otimes \hat{I}_{\mathcal{N}_t} \otimes \hat{I}_{\mathcal{N}_t} + \hat{I}_{\mathcal{N}_t} \otimes \hat{I}_{\mathcal{N}_t} \otimes \hat{H}_{t_3} \otimes \hat{I}_{\mathcal{N}_t} + \hat{I}_{\mathcal{N}_t} \otimes \hat{I}_{\mathcal{N}_t} \otimes \hat{I}_{\mathcal{N}_t} \otimes \hat{H}_{t_4},$$

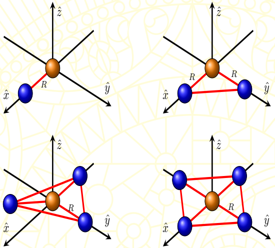
Control-target Hamiltonian:

$$|r\rangle\langle r| \otimes |R\rangle_1\langle R| = |r\rangle\langle r| \otimes \left(|R\rangle_1\langle R| \otimes \hat{I}_{\mathcal{N}_t} \otimes \otimes \hat{I}_{\mathcal{N}_t} \otimes \hat{I}_{\mathcal{N}_t} \right),$$

Target-target Hamiltonian:

$$|R\rangle_1\langle R| \otimes |R\rangle_2\langle R| = \hat{I}_{\mathcal{N}_c} \otimes \left(|R\rangle_1\langle R| \otimes |R\rangle_2\langle R| \otimes \otimes \hat{I}_{\mathcal{N}_t} | \otimes \hat{I}_{\mathcal{N}_t} \right).$$

Spatial configurations of CNOT^{N} gates



$$\hat{\mathcal{H}}_t = \hat{H}_{t_1} \otimes \hat{I}_{\mathcal{N}_t} \otimes \hat{I}_{\mathcal{N}_t} \otimes \hat{I}_{\mathcal{N}_t} + \hat{I}_{\mathcal{N}_t} \otimes \hat{H}_{t_2} \otimes \hat{I}_{\mathcal{N}_t} \otimes \hat{I}_{\mathcal{N}_t} + \hat{I}_{\mathcal{N}_t} \otimes \hat{I}_{\mathcal{N}_t} \otimes \hat{H}_{t_3} \otimes \hat{I}_{\mathcal{N}_t} + \hat{I}_{\mathcal{N}_t} \otimes \hat{I}_{\mathcal{N}_t} \otimes \hat{I}_{\mathcal{N}_t} \otimes \hat{H}_{t_4},$$

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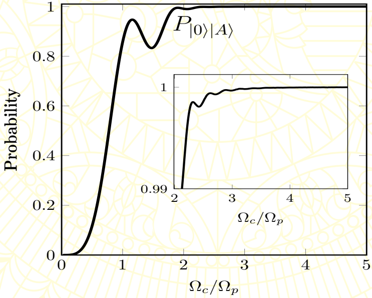
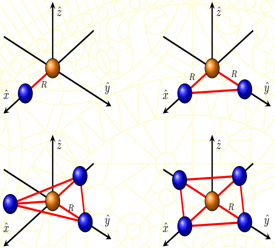
The behavior of blocking gate:-



• $\Delta = 2\pi \times 1.2$ GHz, • $\Omega_p = 2\pi \times 50$ MHz.

Spatial configurations of CNOT^N gates



$$\hat{\mathcal{H}}_t = \hat{H}_{t_1} \otimes \hat{I}_{\mathcal{N}_t} \otimes \hat{I}_{\mathcal{N}_t} \otimes \hat{I}_{\mathcal{N}_t} + \hat{I}_{\mathcal{N}_t} \otimes \hat{H}_{t_2} \otimes \hat{I}_{\mathcal{N}_t} \otimes \hat{I}_{\mathcal{N}_t} + \hat{I}_{\mathcal{N}_t} \otimes \hat{I}_{\mathcal{N}_t} \otimes \hat{H}_{t_3} \otimes \hat{I}_{\mathcal{N}_t} + \hat{I}_{\mathcal{N}_t} \otimes \hat{I}_{\mathcal{N}_t} \otimes \hat{I}_{\mathcal{N}_t} \otimes \hat{H}_{t_4},$$

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Target-target Hamiltonian:

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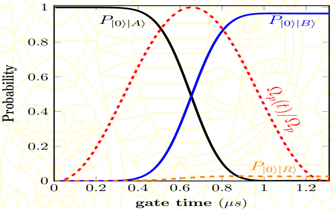

• $\Omega_c = 0.15\,\Omega_p$

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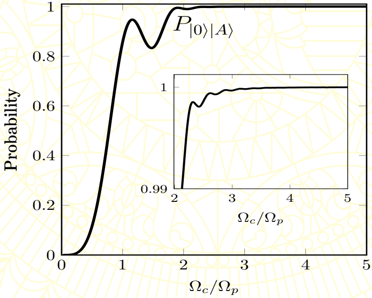
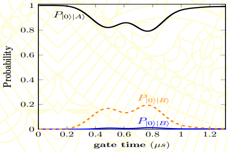
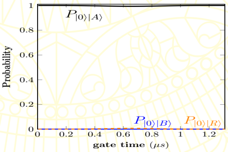


• $\Delta = 2\pi \times 1.2$ GHz, • $\Omega_p = 2\pi \times 50$ MHz.



• $\Omega_c = 2\,\Omega_p$



• $\Omega_c = 14\,\Omega_p$

Interaction energies and Fidelity

These values are calculated using ARC considering $\Delta n = 2$, $\Delta \ell = 2$, $\theta = \pi/2$, $\phi = 0$, minimum state contribution $= 0.2$, and maximum energy difference $\Delta E/h = 25$ GHz.

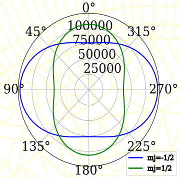
№	Atom 1	Atom 2	R_{LR} (μm)	R_{vdW} (μm)	$C_3/2\pi$ (GHz $\cdot$ μm^3)	$C_6/2\pi$ (GHz $\cdot$ μm^6)
1	Rubidium $\lvert 77S_{1/2}, m_j = 1/2\rangle$	Rubidium $\lvert 77S_{1/2}, m_j = 1/2\rangle$	1.8	4.5	4.20	2036
2	Cesium $\lvert 81S_{1/2}, m_j = -1/2\rangle$	Cesium $\lvert 81S_{1/2}, m_j = -1/2\rangle$	2.0	9.5	1.92	2364
3	Rubidium $\lvert 77S_{1/2}, m_j = 1/2\rangle$	Cesium $\lvert 81S_{1/2}, m_j = -1/2\rangle$	1.9	31.5	14.25	2484

[ARC, Comp. Phys. Com. 220, 319 (2017).]

- R_{LR} is the interatomic distance where the theory of Le Roy-Bernstein is satisfied.
- R_{vdW} is crossover distance, marking boundary between dipole-dipole and vdW regimes.
- C_3 is the dispersive coefficient of d-d interaction energy.
- C_6 is the dispersive coefficient of vdW interaction energy.

Interaction energies and Fidelity

These values are calculated using ARC considering $\Delta n = 2$, $\Delta \ell = 2$, $\theta = \pi/2$, $\phi = 0$, minimum state contribution $= 0.2$, and maximum energy difference $\Delta E/h = 25$ GHz.



№	Atom 1	Atom 2	R_{LR} (μm)	R_{vdW} (μm)	$C_3/2\pi$ (GHz $\cdot$ μm^3)	$C_6/2\pi$ (GHz $\cdot$ μm^6)
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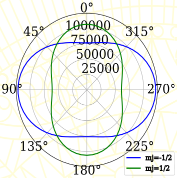
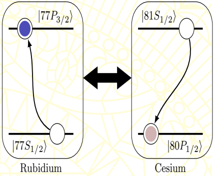


Dominant interaction channel of heteronuclear interaction



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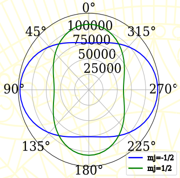

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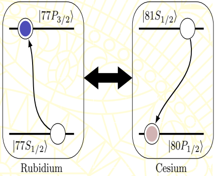

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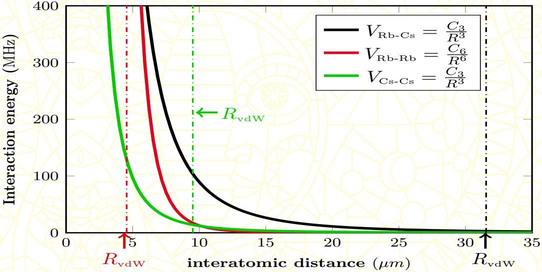

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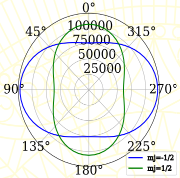

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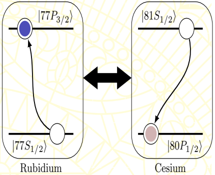

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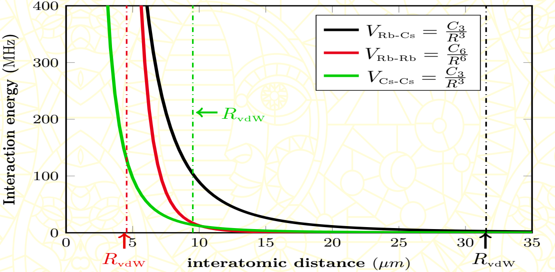

Fidelity

$$F = \text{Tr}\left(\sqrt{\sqrt{\hat{\rho}}\hat{\sigma}\sqrt{\hat{\rho}}}\right)$$

$\hat{\rho}$ is the density matrix of the computational basis

$$\frac{1}{\sqrt{2}}\left(|0\rangle|A\rangle^{\otimes N} + (|1\rangle|A\rangle^{\otimes N}\right) \rightarrow \frac{1}{\sqrt{2}}\left(|0\rangle|A\rangle^{\otimes N} + (|1\rangle|B\rangle^{\otimes N}\right)$$

$$\hat{\sigma} = |\Phi^+\rangle\langle\Phi^+|$$

$|\Phi^+\rangle$ is one of Bell states basis.

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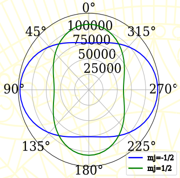

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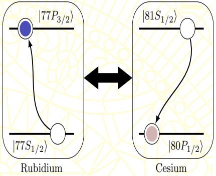

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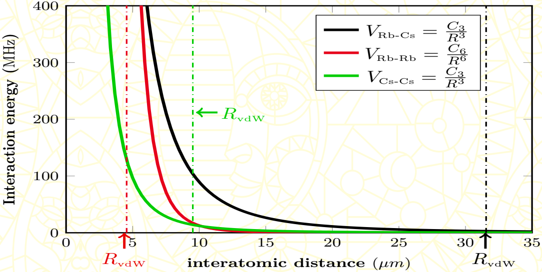

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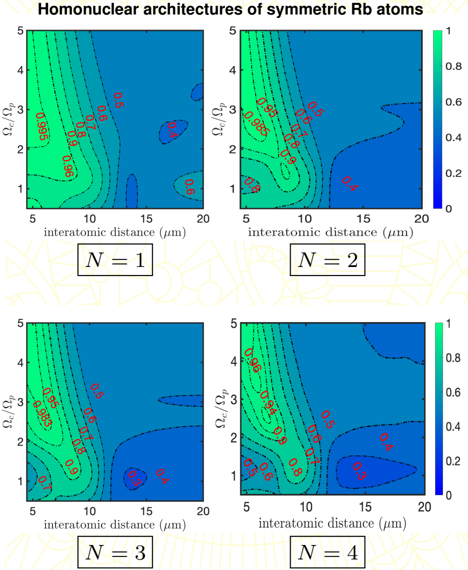
$$\hat\sigma = |\Phi^+\rangle\langle\Phi^+|$$

$|\Phi^+\rangle$ is one of Bell states basis.

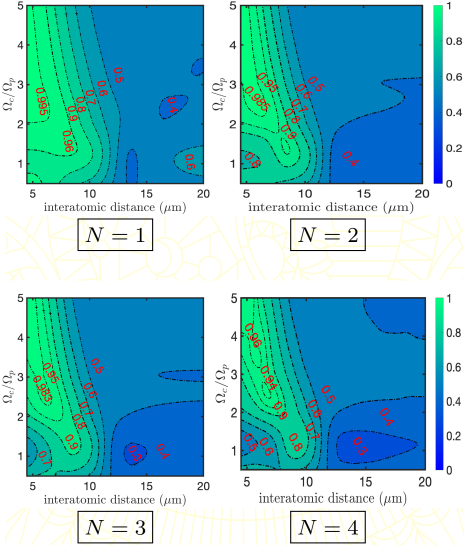
- $\Delta = 2\pi \times 1.2$ GHz,
- $\Omega_p = 2\pi \times 50$ MHz,
- $\tau_{|r\rangle} = 505 - 548\ \mu s$,
- $|P\rangle = |6\ P_{3/2}\rangle$.
- $\tau_{|P\rangle} = 0.131\ \mu s$.

Fidelity contours

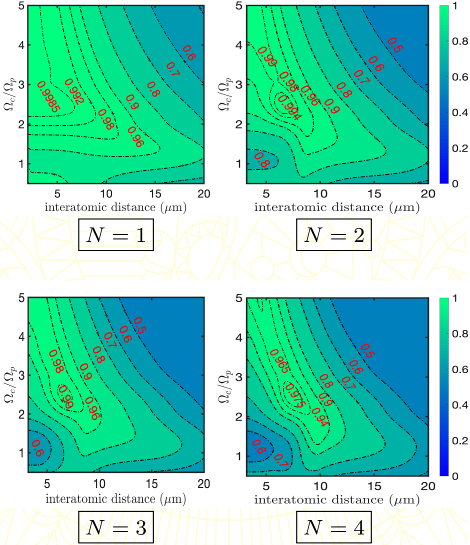


Fidelity contours

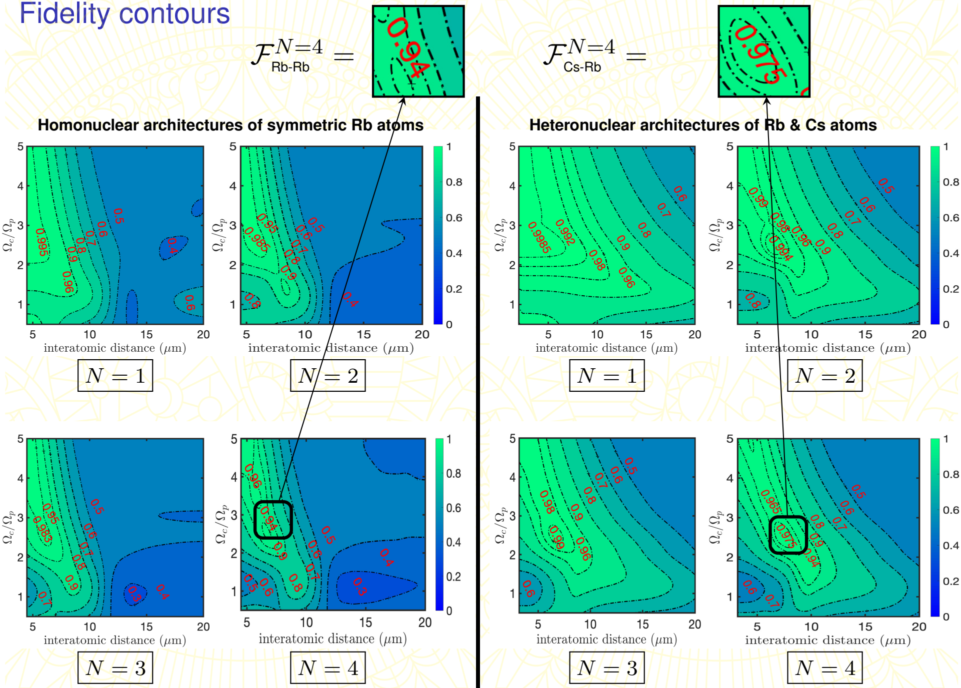
Homonuclear architectures of symmetric Rb atoms



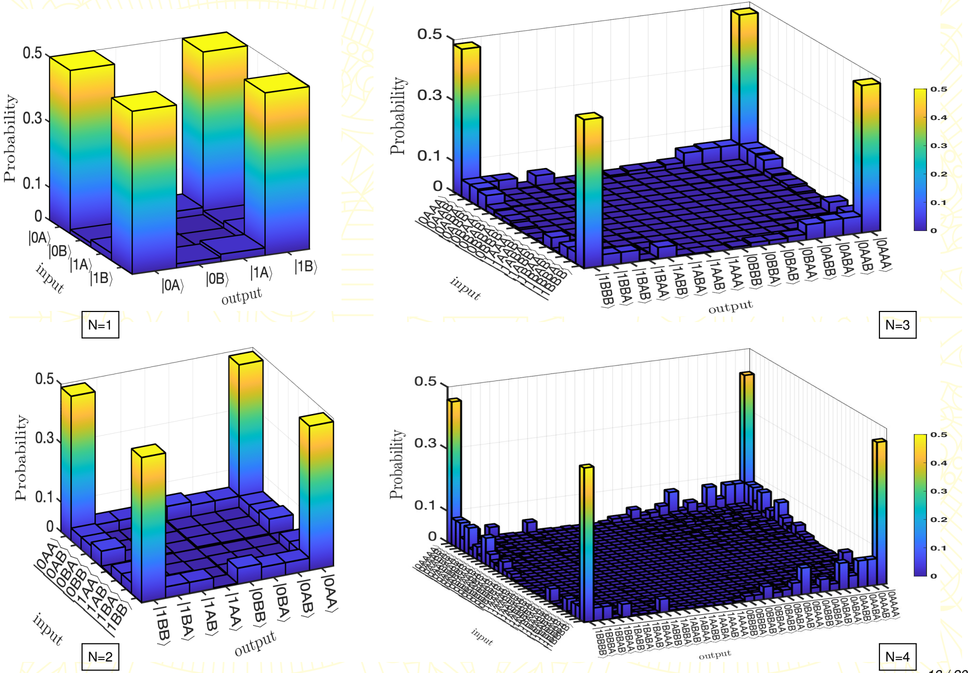
Heteronuclear architectures of Rb & Cs atoms



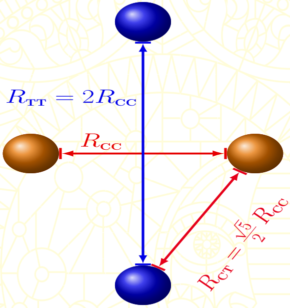
Fidelity contours

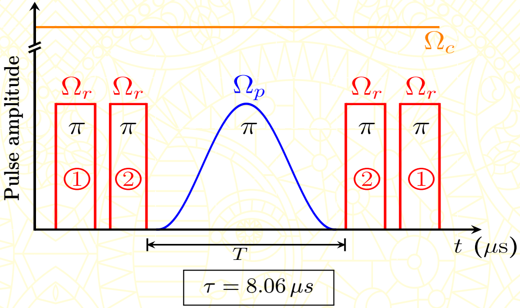


Generation of GHZ-state



Demonstrating $\mathbf{C_2NOT^2}$ gates



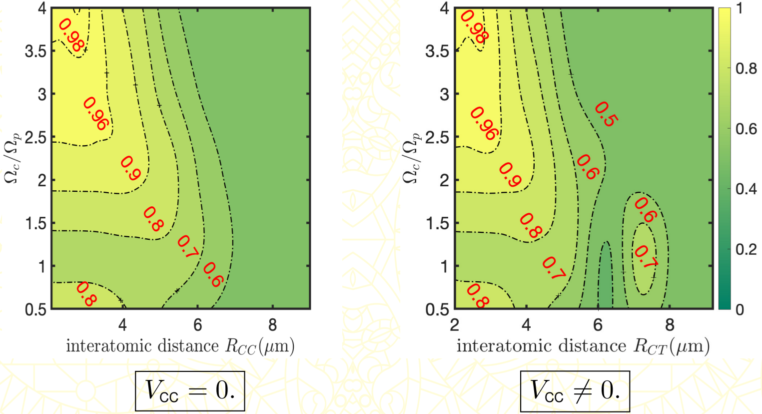


(I) Blocking:
$$|0\,0\rangle|A\,A\rangle \rightarrow |0\,0\rangle|A\,A\rangle,$$
$$|0\,0\rangle|B\,B\rangle \rightarrow |0\,0\rangle|B,B\rangle,$$
$$|0\,0\rangle|A\,B\rangle \rightarrow |0\,0\rangle|A\,B\rangle,$$
$$|0\,0\rangle|B\,A\rangle \rightarrow |0\,0\rangle|B\,A\rangle,$$

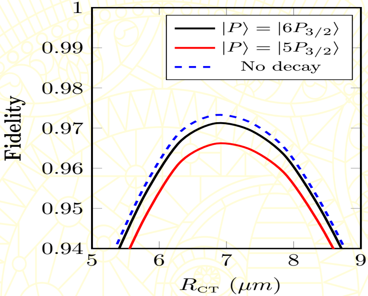
(II)Transfer:
$$|1\,1\rangle|A\,A\rangle \rightarrow |1\,1\rangle|B\,B\rangle,$$
$$|1\,1\rangle|B\,B\rangle \rightarrow |1\,1\rangle|A\,A\rangle,$$
$$|0\,1\rangle|A\,B\rangle \rightarrow |0\,1\rangle|B\,A\rangle,$$
$$|0\,1\rangle|B\,A\rangle \rightarrow |0\,1\rangle|A\,B\rangle,$$
$$|1\,0\rangle|A\,B\rangle \rightarrow |1\,0\rangle|B\,A\rangle,$$
$$|1\,0\rangle|B\,A\rangle \rightarrow |1\,0\rangle|A\,B\rangle.$$

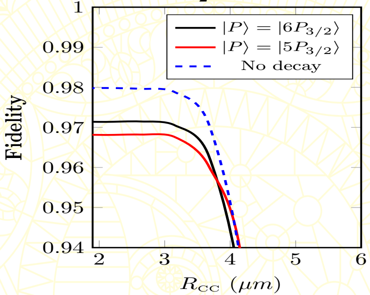
Fidelity of C_2NOT^2

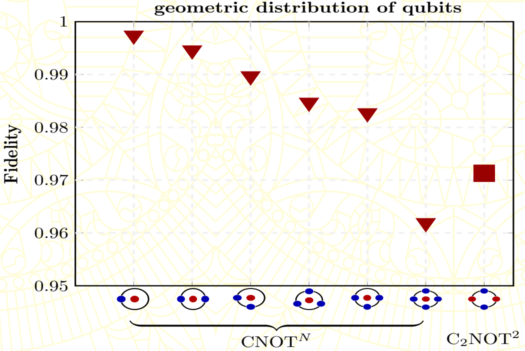
- Interaction of cesium atoms as control and rubidium atoms as targets.



Gate Errors | Heteronuclear architectures of Cs control atom(s) and Rb target atoms

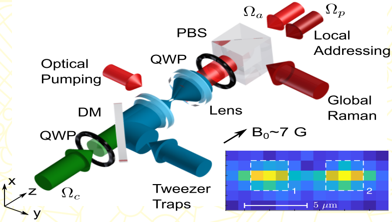


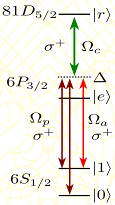




Foremost Experimental work demonstrating EIT

McDonnell et al, PRL 129, 200501 (2022)
ArXiV:2204.03733 (2022)





Experiment Setup

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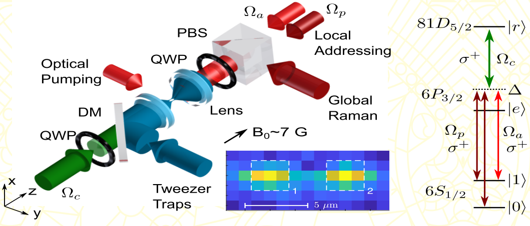

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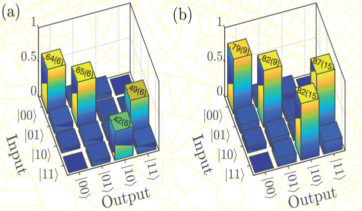

(a) $\mathcal{F}^{\text{RAW}} = 0.55$ (b) $\mathcal{F}^{\text{cor}} = 0.82$

Gate Measurement

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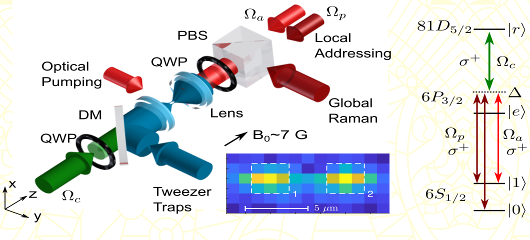

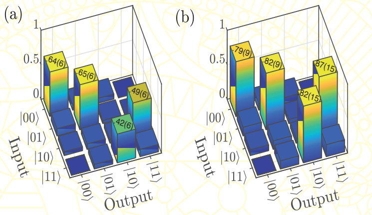

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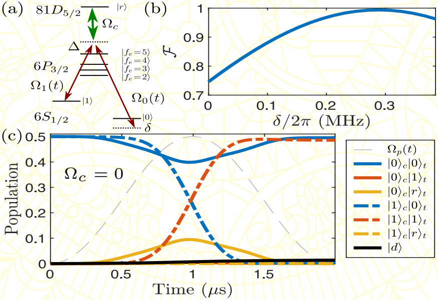

EIT Protocol via $6\,P_{3/2}$

$\Delta/2\pi = 1.05$GHz.
$\mathcal{F} = 0.98$

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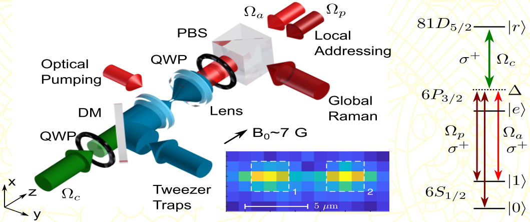

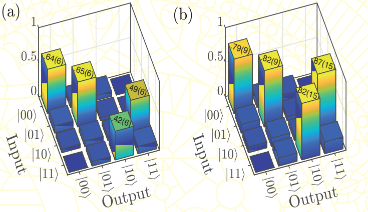

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Experiment Setup

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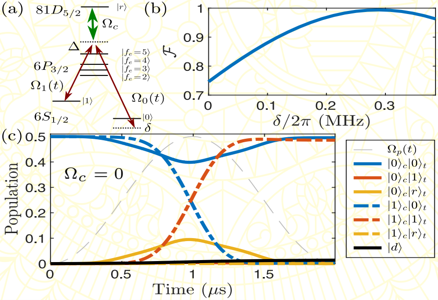

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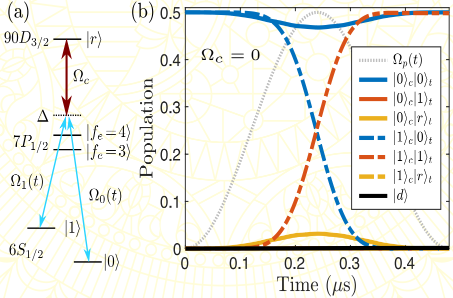
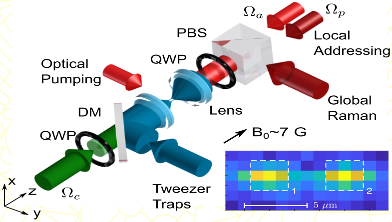


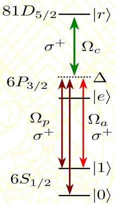
EIT Protocol via $7\,P_{1/2}$

$\Delta/2\pi = 5$GHz.
$\mathcal{F} > 0.998$

Foremost Experimental work demonstrating EIT

McDonnell et al. PRL 129, 200501 (2022)
ArXiV:2204.03733 (2022)





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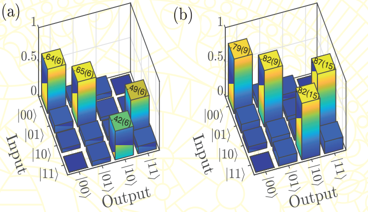

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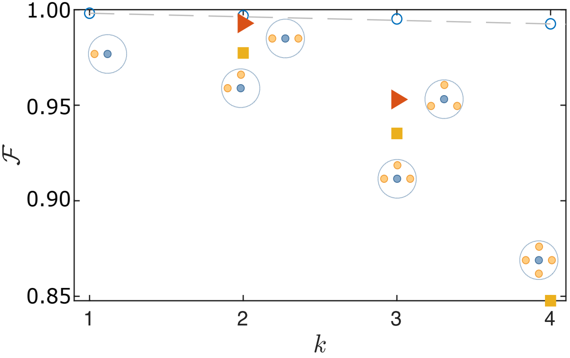
Gate Measurement



Conclusion and Outlook

We can conclude the following:

- Heteronuclear architecture of alkali atoms is much better than the homonuclear architecture in terms of fidelity.
- The obtained Fidelity for CNOT^4, with cesium control atom and rubidium target atoms, is $\simeq 97.5\%$ for fast gate implementation $\tau = 1.303\ \mu$s and moderate value of Rabi frequency $\Omega_c > 2\pi \times 125$ MHz.
- The obtained fidelity is higher than the lower bound fidelity threshold for creating logic qubits of CNOT gates without the need to perform error correction surface codes.
  - **IEEE Access, 7, 121501-121529, (2019).**
- Using the 2^{nd} resonance levels of target atoms (for Rb $5\,S_{1/2} \rightarrow 6\,P_{3/2} \rightarrow n\,S_{1/2}$, and for Cs $6\,S_{1/2} \rightarrow 7\,P_{3/2} \rightarrow n'\,S_{1/2}$) enhances the fidelity of the combined system.

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- The obtained fidelity is higher than the lower bound fidelity threshold for creating logic qubits of CNOT gates without the need to perform error correction surface codes.
  - **IEEE Access, 7, 121501-121529, (2019).**
- Using the 2^{nd} resonance levels of target atoms (for Rb $5\,S_{1/2} \rightarrow 6\,P_{3/2} \rightarrow n\,S_{1/2}$, and for Cs $6\,S_{1/2} \rightarrow 7\,P_{3/2} \rightarrow n'\,S_{1/2}$) enhances the fidelity of the combined system.

Future work

- Study the effect of coherent transport of a moving control atom implementing CNOT^{N} gate and the mutual information of the subsystems.

End of text!