Сдвиг частоты в оптических атомных часах, обусловленный неконтролируемой эллиптичностью пробного поля

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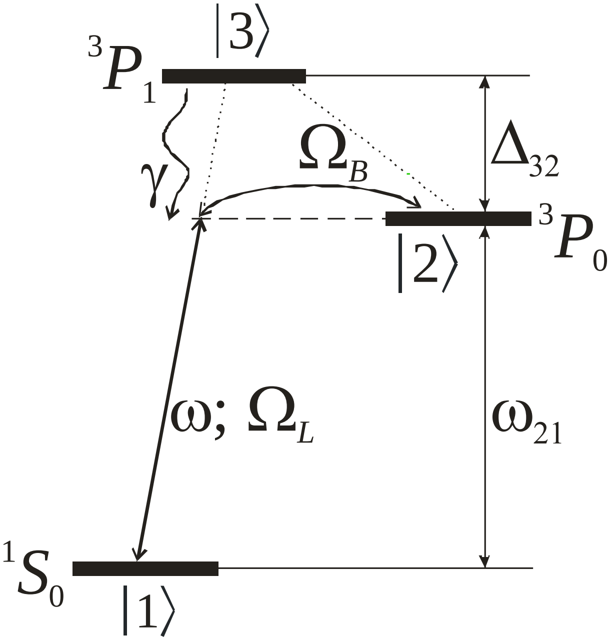
Motivation of our investigations

Ultra-precise atomic clocks are the forefront of modern quantum sensors and tests of the standard model. At present, some laboratories have demonstrated systematic uncertainties and long-term instabilities at a fractional level of 10^{-18} both for devices with neutral atoms trapped in an optical lattice at the magic wavelength, and for clocks using trapped ions. There is a recent trend to push fractional uncertainties to the level of 10^{-19}. However, such an extraordinary high metrological precision requires a very thorough study of all possible frequency shifts that can exceed (at least in principle) the value of 10^{-19}.
We consider the ac Stark shift due to some small residual ellipticity of the probe field, which has not previously been systematically investigated.

The method of magnetically induced excitation of forbidden transition $^1S_0 \rightarrow {}^3P_0$ for even isotopes (with zero nuclear spin)



For mixing the state $^3P_1$ with the state $^3P_1$ we use an external static magnetic field.
Here the transition $^1S_0 \rightarrow {}^3P_0$ becomes partially resolved and can be excited by usual one-photon method with a single probe laser. Probability of the transition is ruled by the value of magnetic field.

Under using of the probe laser field $\mathbf{E}e^{-i\omega t}$ with frequency of the transition $^1S_0 \rightarrow {}^3P_0$ ($\omega = \omega_{21}$) the effective Rabi frequency V_{12} is equal to:

$$|2\rangle \rightarrow |2\rangle + (\Omega_B / \Delta_{32})|3\rangle$$

$$V_{12} = \frac{\Omega_L\, \Omega_B}{\Delta_{32}}$$

$\Omega_L = \langle 3|\hat{\mathbf{d}} \cdot \mathbf{E}|1\rangle/\hbar$ is Rabi frequency of the probe field via transition $^1S_0 \rightarrow {}^3P_1$

$\Omega_B = \langle 2|\hat{\mu} \cdot \mathbf{B}|3\rangle/\hbar$, is matrix element of magneto-induced interaction on transition $^3P_1 \rightarrow {}^3P_0$

E is vector amplitude of probe laser field;
B is vector of static magnetic field.

$$V_{12} = \frac{\langle\|d\|\rangle\, \langle\|\mu\|\rangle(\mathbf{E}\cdot\mathbf{B})}{\hbar^2 \Delta_{32}}$$

$\langle\|\mu\|\rangle = \sqrt{2/3}\,\mu_B$ for alkali-earth-like atoms

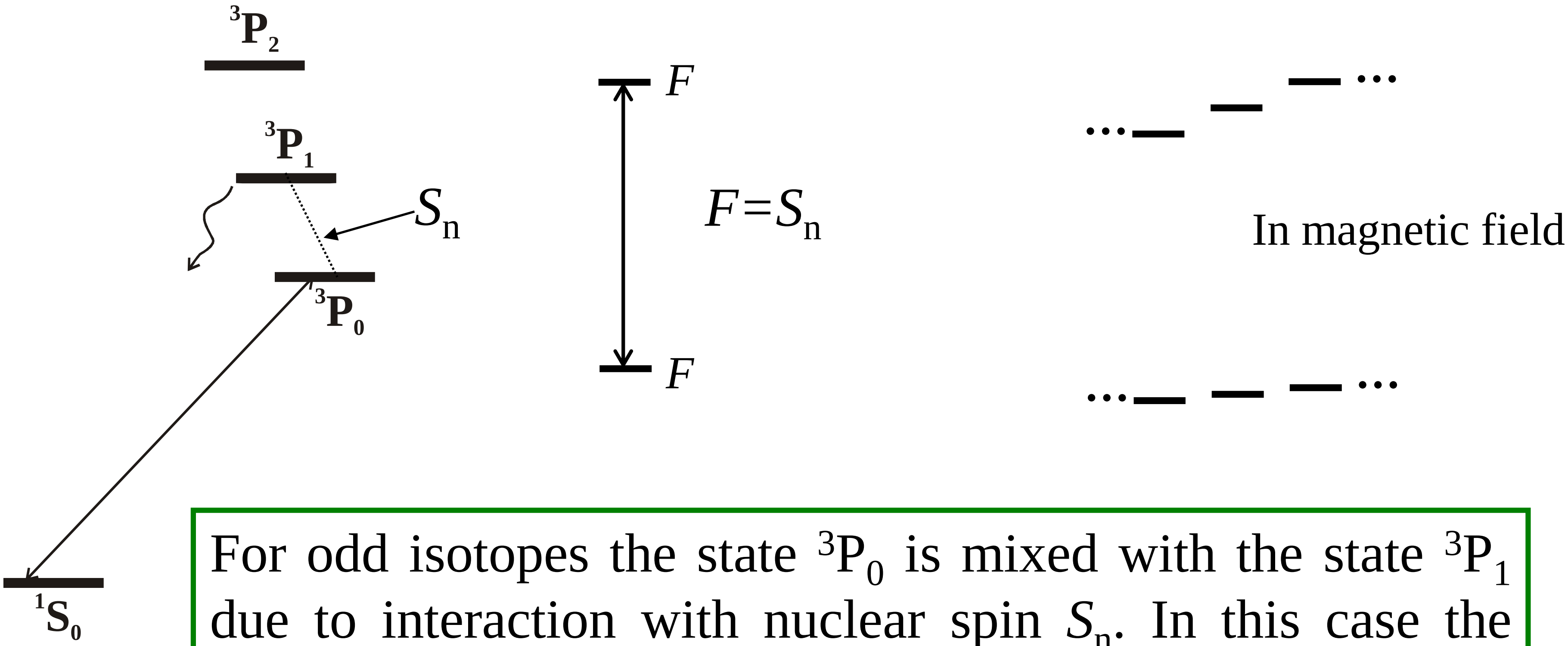
The method of magnetically induced excitation of forbidden transition $^1S_0 \rightarrow {}^3P_0$ for even isotopes (with zero nuclear spin)

Because the used magnetic field is relatively small (at the level a few mT), we should use relatively strong probe field intensity (at the level 10-1000 mW/cm^2) to produce a convenient (for real experiments) Rabi frequency at the level 1-10 Hz.

In this case the ac-Stark shift produced only by other (very far-off-resonant transitions) becomes quite large (at the level 10-100 Hz). Therefore we need to carefully control this shift, or to develop new spectroscopic methods (for example, hyper-Ramsey method) to suppress these shifts.

However, for odd isotopes the situation is another.

Excitation of strongly forbidden transition $^1S_0 \rightarrow {}^3P_0$ for odd isotopes (nuclear spin is non-zero)



For odd isotopes the state $^3P_0$ is mixed with the state $^3P_1$ due to interaction with nuclear spin S_n. In this case the transition $^1S_0 \rightarrow {}^3P_0$ becomes partially resolved (natural width ~mHz). And we can excite this transition by usual one-photon method with small intensities of probe laser. The same energy picture we have for some ion clock (e.g., for $^{27}Al^+$, $^{115}In^+$)

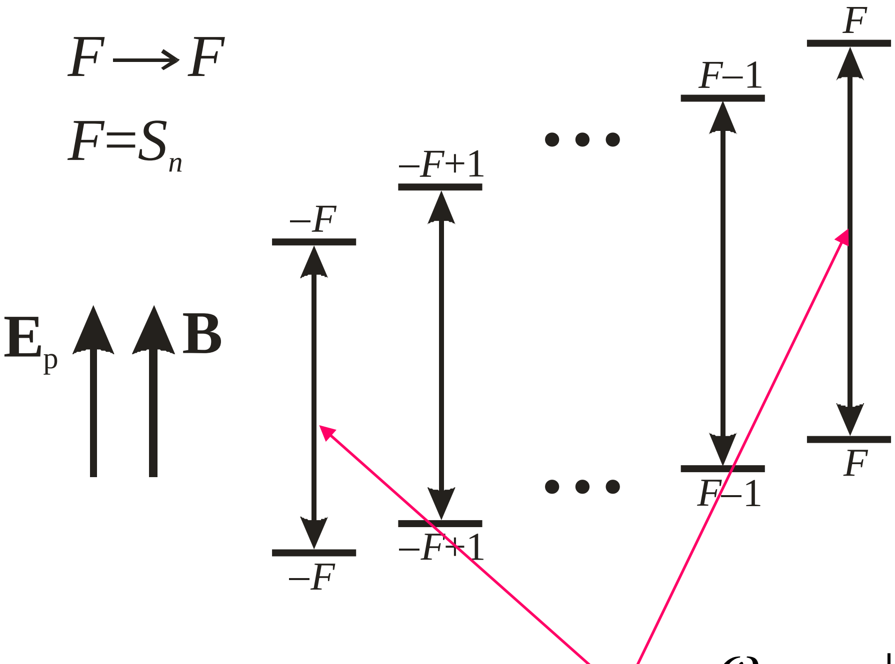
Excitation of strongly forbidden transition $^1S_0 \rightarrow {}^3P_0$ for odd isotopes (nuclear spin is non-zero)

Thus, for odd isotopes we can use much smaller intensities (by the orders of 10^{-5}-10^{-6}) of the probe laser in comparison with even isotopes. In this case the as-Stark shift produced by far-off-resonant transitions becomes very small.

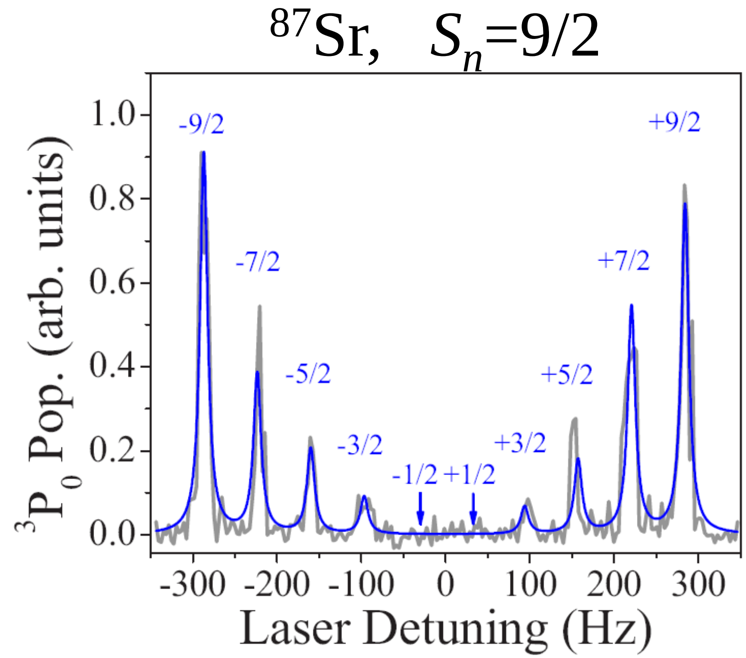
Probably namely this reason produces an opinion that the probe field-induced ac-Stark shifts for odd isotopes is not a problem at all (i.e. automatically). But in reality the situation is not so trivial.

Why?

Clock transition $F \rightarrow F$ ($^1S_0 \rightarrow {}^3P_0$) spectroscopy for odd isotopes in the linear polarized probe field



$$\omega_{\text{clock}} = \frac{\omega_{-F-F} + \omega_{FF}}{2}$$



M.M.Boyd et al., PRA **76**, 022510 (2007)

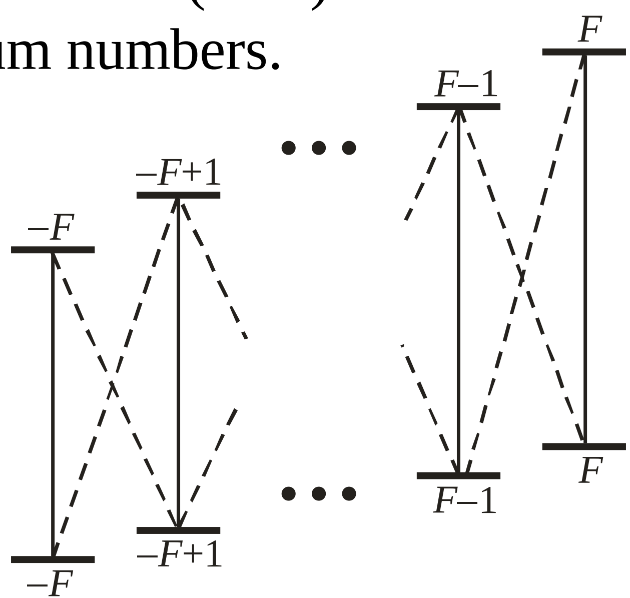
Thus, we measure the frequencies $\omega_{m\text{-}m}$ of transitions *m-m*, each of them is shifted by the magnetic field **B**. But, the clock transition frequency ω_{clock} is not shifted by **B**.

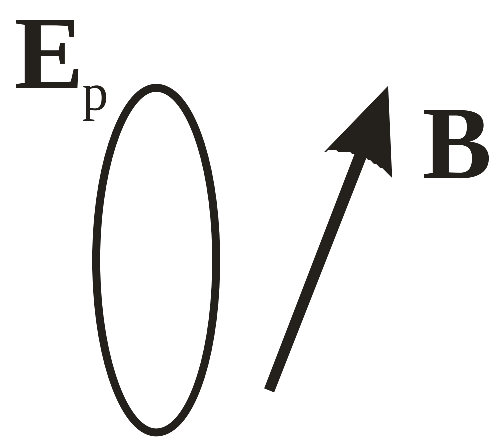
In the ideal case $\mathbf{B} \| \mathbf{E}_p$ an additional probe field-induced shifts are absent.

Clock transition ($^1S_0 \rightarrow {}^3P_0$) spectroscopy for odd isotopes for the non-ideal polarization conditions

But, in reality we never have the ideal polarization condition $\mathbf{B} \| \mathbf{E}_p$.

In our case the non-ideality is due to both some *uncontrolled ellipticity* of the probe field vector $\mathbf{E}_p$ and some *uncontrolled orientation* of the magnetic field **B**. Thus, for the careful investigation of the possible additional shifts firstly we should consider the general case of the elliptically polarized probe field $\mathbf{E}_p$ and arbitrary oriented magnetic field vector **B**. In this general case apart of linear π-component we must take into account other two circular $\sigma_{\pm}$-components of the probe field vector $\mathbf{E}_p$. These components induce other transitions *m*-(*m*±1) between Zeeman sublevels with different magnetic quantum numbers.





Near-resonant probe field ellipticity-induced ac-Stark shifts of the clock transition ($^1S_0 \to {}^3P_0$) for odd isotopes

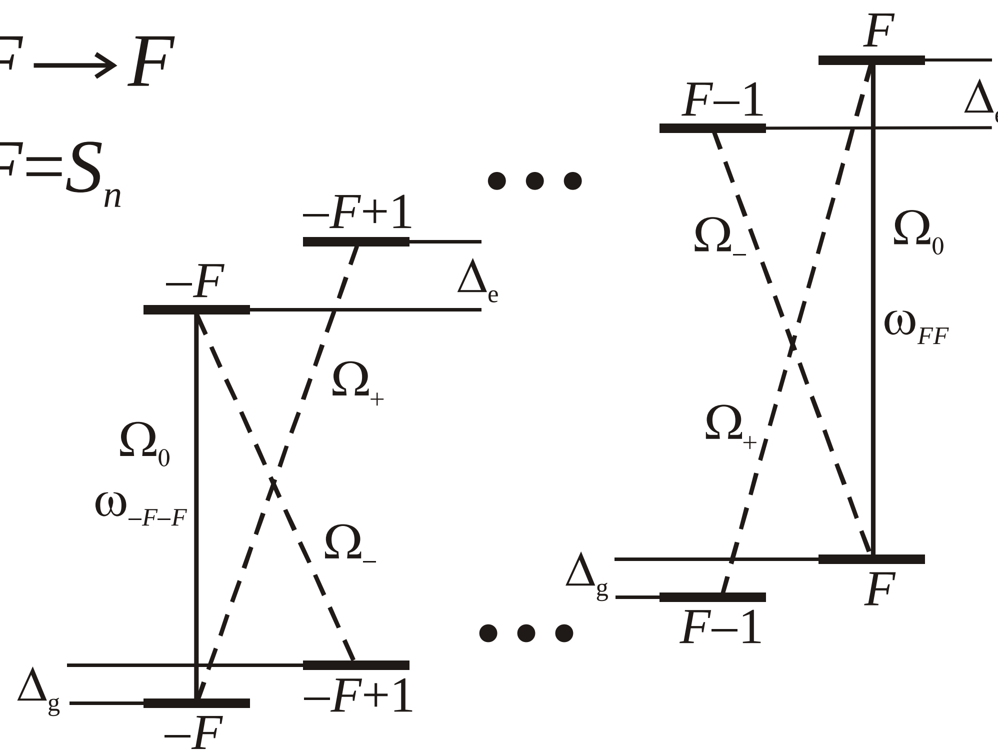
Due to circular components $\sigma_\pm$ each of frequencies ω_{mm} has additional ac-Stark shift. In particular, the end frequencies ω_{-F-F} and ω_{FF} have the following shifts:

$$\bar{\delta}_{-F-F} = \frac{|\Omega_+|^2}{4\Delta_\mathrm{e}} - \frac{|\Omega_-|^2}{4\Delta_\mathrm{g}},$$

$$\bar{\delta}_{+F+F} = \frac{|\Omega_+|^2}{4\Delta_\mathrm{g}} - \frac{|\Omega_-|^2}{4\Delta_\mathrm{e}},$$

$F \longrightarrow F$

$F = S_n$



$$\bar{\delta}_\mathrm{ac}^\mathrm{(el\text{-}ind)} = \frac{\bar{\delta}_{-F-F} + \bar{\delta}_{+F+F}}{2}$$

Thus, the measured clock frequency $\omega'_\mathrm{clock} = (\omega'_{FF} + \omega'_{-F-F})/2$ also will be shifted! $\Rightarrow$

$$= \frac{\Delta_\mathrm{g} + \Delta_\mathrm{e}}{8\Delta_\mathrm{g}\Delta_\mathrm{e}}\left(|\Omega_+|^2 - |\Omega_-|^2\right)$$

Invariant expression for the near-resonant probe field-induced ac-Stark shift of the clock transition ($^1S_0 \rightarrow {}^3P_0$) in odd isotopes

$$\omega_{\text{clock}} = \omega_0 + \bar{\delta}_{\text{ac}}^{(\text{el-ind})}$$

$$\bar{\delta}_{\text{ac}}^{(\text{el-ind})} = \frac{\Delta_{\text{g}} + \Delta_{\text{e}}}{8\Delta_{\text{g}}\Delta_{\text{e}}}\left(|\Omega_+|^2 - |\Omega_-|^2\right)$$

Rabi frequencies $\Omega_\pm$ can be expressed as:

$$\Omega_+ = \frac{\langle d \rangle E_+}{\sqrt{(F+1)}}; \ \Omega_- = \frac{\langle d \rangle E_-}{\sqrt{(F+1)}}$$

Zeeman splittings have a form:

$$\Delta_{\text{g}} = g_{\text{g}} \mu_{\text{B}} |\mathbf{B}|; \ \Delta_{\text{e}} = g_{\text{e}} \mu_{\text{B}} |\mathbf{B}|$$

It allows us to express the shift as following:

$$\bar{\delta}_{\text{ac}}^{(\text{el-ind})} = \left|\langle d \rangle\right|^2 \frac{(g_{\text{g}} + g_{\text{e}})}{8 g_{\text{g}} g_{\text{e}}} \frac{\left(|E_+|^2 - |E_-|^2\right)}{(F+1)\mu_{\text{B}} |\mathbf{B}|}$$

It can be rewritten in the invariant vector form:

$$\bar{\delta}_{\text{ac}}^{(\text{el-ind})} = -\left|\langle d \rangle\right|^2 \frac{(g_{\text{g}} + g_{\text{e}})}{8 g_{\text{g}} g_{\text{e}}} \frac{(i[\mathbf{E}_{\text{p}}^* \times \mathbf{E}_{\text{p}}] \cdot \mathbf{n}_{\mathbf{B}})}{\mu_{\text{B}} |\mathbf{B}| (F+1)}; \text{ where } \mathbf{n}_{\mathbf{B}} = \frac{\mathbf{B}}{|\mathbf{B}|}$$

Invariant expression for the near-resonant probe field ellipticity-induced ac-Stark shift (continuation 2)

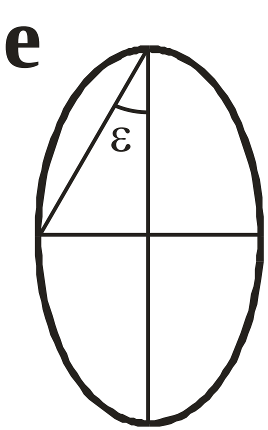
$$\bar{\delta}_{\mathrm{ac}}^{(\text{el-ind})} = -\left|\langle d \rangle\right|^2 \frac{(g_\mathrm{g} + g_\mathrm{e})}{8 g_\mathrm{g} g_\mathrm{e}} \frac{(i[\mathbf{E}_\mathrm{p}^* \times \mathbf{E}_\mathrm{p}] \cdot \mathbf{n_B})}{\mu_\mathrm{B} |\mathbf{B}| (F+1)}; \text{ where } \mathbf{n_B} = \frac{\mathbf{B}}{|\mathbf{B}|}$$

We can express the probe field vector $\mathbf{E}_\mathrm{p}$ as following:

$$\mathbf{E}_\mathrm{p} = E\mathbf{e}$$

where **e** is unit vector of elliptical polarization, which has the form:

$$\mathbf{e} = \cos(\varepsilon)\mathbf{e}_x + i\sin(\varepsilon)\mathbf{e}_y$$

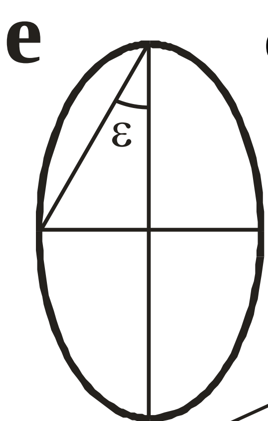


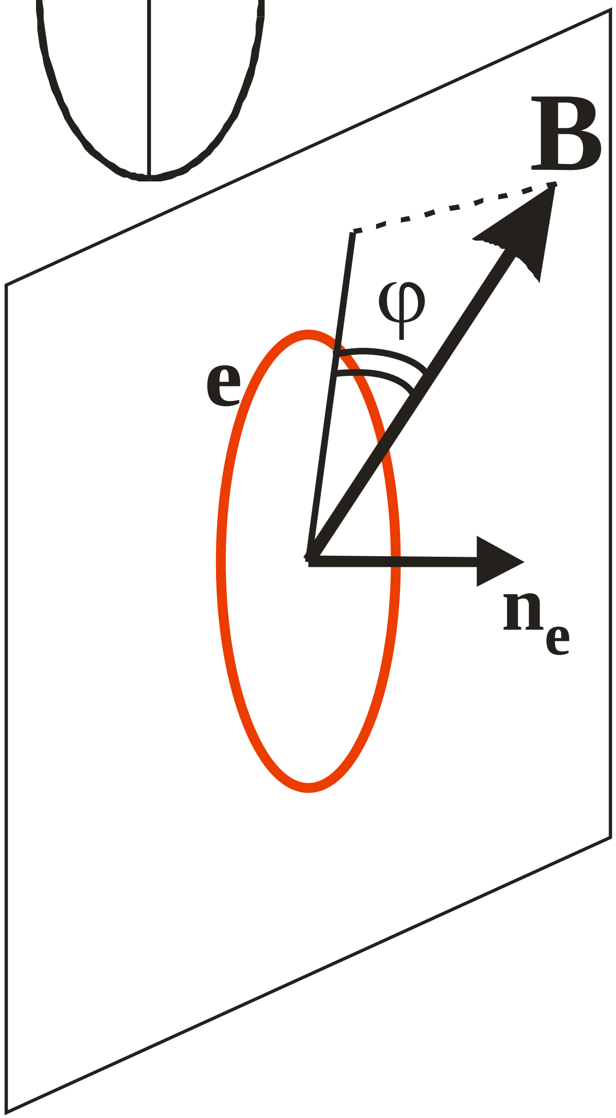
$$\bar{\delta}_{\mathrm{ac}}^{(\text{el-ind})} = -\left|\langle d \rangle E\right|^2 \frac{(g_\mathrm{g} + g_\mathrm{e})}{8 g_\mathrm{g} g_\mathrm{e}} \frac{(i[\mathbf{e}^* \times \mathbf{e}] \cdot \mathbf{n_B})}{\mu_\mathrm{B} |\mathbf{B}| (F+1)}$$

$$i[\mathbf{e}^* \times \mathbf{e}] = \sin(2\varepsilon)\mathbf{n_e}$$

where $\mathbf{n_e}$ is a unit vector, which is orthogonal to the plane of elliptical polarization **e**.

Invariant expression for the near-resonant probe field ellipticity-induced ac-Stark shift (continuation 3)





$$\bar{\delta}_{\rm ac}^{(\rm el\text{-}ind)} = -\left|\langle d\rangle E\right|^2 \frac{(g_{\rm g}+g_{\rm e})}{8g_{\rm g}g_{\rm e}}\frac{\sin(2\varepsilon)(\mathbf{n_e}\cdot\mathbf{n_B})}{\mu_{\rm B}\,|\mathbf{B}|\,(F+1)}$$

$$(\mathbf{n_e}\cdot\mathbf{n_B}) = \sin(\varphi)$$

where φ is the angle between vector **B** and plane of elliptical polarization **e**. Thus, we have the following convenient expression:

$$\bar{\delta}_{\rm ac}^{(\rm el\text{-}ind)} = -\left|\langle d\rangle E\right|^2 \frac{(g_{\rm g}+g_{\rm e})}{8g_{\rm g}g_{\rm e}}\frac{\sin(2\varepsilon)\sin(\varphi)}{\mu_{\rm B}\,|\mathbf{B}|\,(F+1)}$$

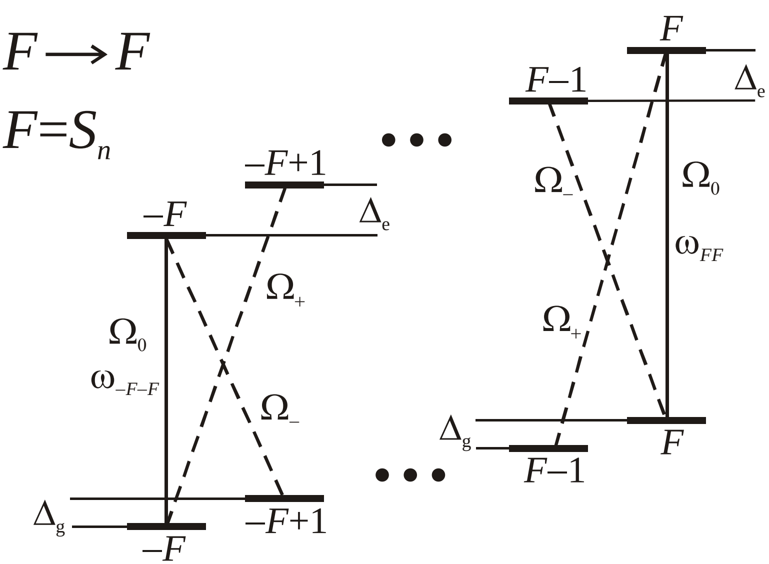
This shift is equal to zero if:

1. For linear polarization ε=0 (φ is arbitrary).
2. For φ=0, i.e. if **B** lies in the plane **e** (ε is arbitrary).
3. In “exotic” case $g_g = -g_e$ for arbitrary ε and φ .

Metrological estimations of the near-resonant probe field ellipticity-induced ac-Stark shift

$$|\bar{\delta}_{\text{ac}}^{(\text{el-ind})}| = \frac{|g_{\text{g}} + g_{\text{e}}|}{8|g_{\text{g}} g_{\text{e}}|} |\sin(2\varepsilon)\sin(\varphi)| \frac{|\langle d \rangle E|^2}{\mu_{\text{B}} |\mathbf{B}| (F+1)}$$

With respect to the real experiments we should assume that $|\varepsilon,\varphi| << 1$. In this case we can use the following approximation:

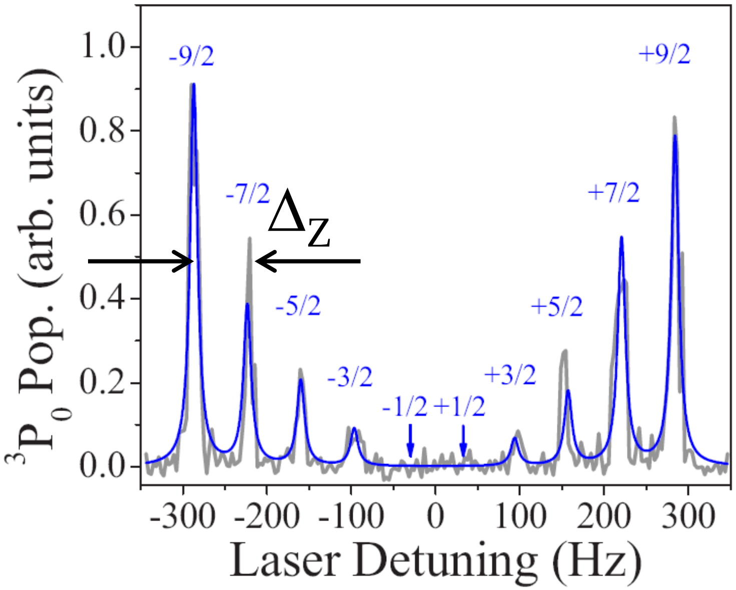


$$|\bar{\delta}_{\text{ac}}^{(\text{el-ind})}| \approx \frac{|g_{\text{g}} + g_{\text{e}}|}{4\ |g_{\text{g}} g_{\text{e}}|} |\varepsilon\varphi| \frac{|\langle d \rangle E|^2}{\mu_{\text{B}} |\mathbf{B}| (F+1)}$$

Moreover, because the magnetic vector **B** is oriented very near to the main axis of elliptical polarization **e**, we can use the approximation: $\Omega_0 \approx \langle d \rangle E \times [F/(F+1)]^{1/2}$.

$$|\bar{\delta}_{\text{ac}}^{(\text{el-ind})}| \approx \frac{|g_{\text{g}} + g_{\text{e}}|}{4|g_{\text{g}} g_{\text{e}}| F} |\varepsilon\varphi| \frac{|\Omega_0|^2}{\mu_{\text{B}} |\mathbf{B}|}$$

Metrological estimations of the near-resonant probe field-induced ac-Stark shift (continuation 2)



$$|\bar{\delta}_{\mathrm{ac}}^{(\mathrm{el\text{-}ind})}| \approx \frac{|g_{\mathrm{g}} + g_{\mathrm{e}}|}{4|g_{\mathrm{g}} g_{\mathrm{e}}|F} |\varepsilon\varphi| \frac{|\Omega_0|^2}{\mu_{\mathrm{B}} |\mathbf{B}|}$$

Also the distance between resonances for the linear polarized probe field can be expressed as $\Delta_Z = |g_e - g_g| \mu_B |\mathbf{B}|$.
It leads to the following final equation, which is expressed (for simplicity) with using experimental parameters Ω_0 and Δ_Z:

$$|\bar{\delta}_{\mathrm{ac}}^{(\mathrm{el\text{-}ind})}| \approx A |\varepsilon\varphi| \frac{|\Omega_0|^2}{\Delta_Z}$$

where the coefficient A is an individual characteristic of given isotope:

$$A = \frac{|g_{\mathrm{g}}^2 - g_{\mathrm{e}}^2|}{4F|g_{\mathrm{g}} g_{\mathrm{e}}|}$$

Metrological estimations of the near-resonant probe field ellipticity-induced ac-Stark shift (continuation 3)

$$|\bar{\delta}_{\rm ac}^{(\rm el\text{-}ind)}| \approx A|\varepsilon\varphi|\frac{|\Omega_0|^2}{\Delta_Z}$$

$$A = \frac{|g_{\rm g}^2 - g_{\rm e}^2|}{4F|g_{\rm g}g_{\rm e}|}$$

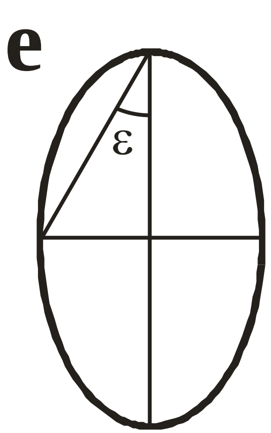
Note, from expression for A we can see that the isotopes with small nuclear spin S_n (in our case $F=S_n$) have an increased probe field-induced as-Stark shift (see ^{171}Yb, which has a most value A).

	^{171}Yb	^{173}Yb	^{87}Sr	^{115}In$^+$	^{27}Al$^+$
F	1/2	5/2	9/2	9/2	5/2
A	0.85	0.18	0.053	0.209	0.045

$$\frac{I_{x'}}{I_{y'}} = \tan^2(\varepsilon) \approx \varepsilon^2 \sim 0.01\,, \;\Rightarrow\; |\varepsilon| \sim 0.1$$

$$|\xi| \sim 0.1$$

$$|\varepsilon\xi| \sim 0.01$$



For typical experimental conditions with lattice-based clocks we estimate the ratio $(|\Omega_0|^2/\Delta_Z)$~0.1-10 Hz, and the value $|\varepsilon\varphi|\sim10^{-2}$. In this case we find that the probe field ellipticity-induced ac-Stark shift can have the relative level of order of 10^{-17}-10^{-19} (especially for ^{171}Yb).

TOTAL AC STARK SHIFT

Since the presented ellipticity-induced shift is proportional to the square of the electric field, $\bar{\delta}_{ac}^{(el-ind)} \propto |E|^2$, it can be considered as an additional, transition internal ac Stark shift. Simultaneously, there always exists the well-known standard ac Stark shift, $\bar{\delta}_{ac}^{(off-res)} \propto |E|^2$, due to the interaction of the probe field with other far-off-resonant atomic levels, and which can formally be represented as

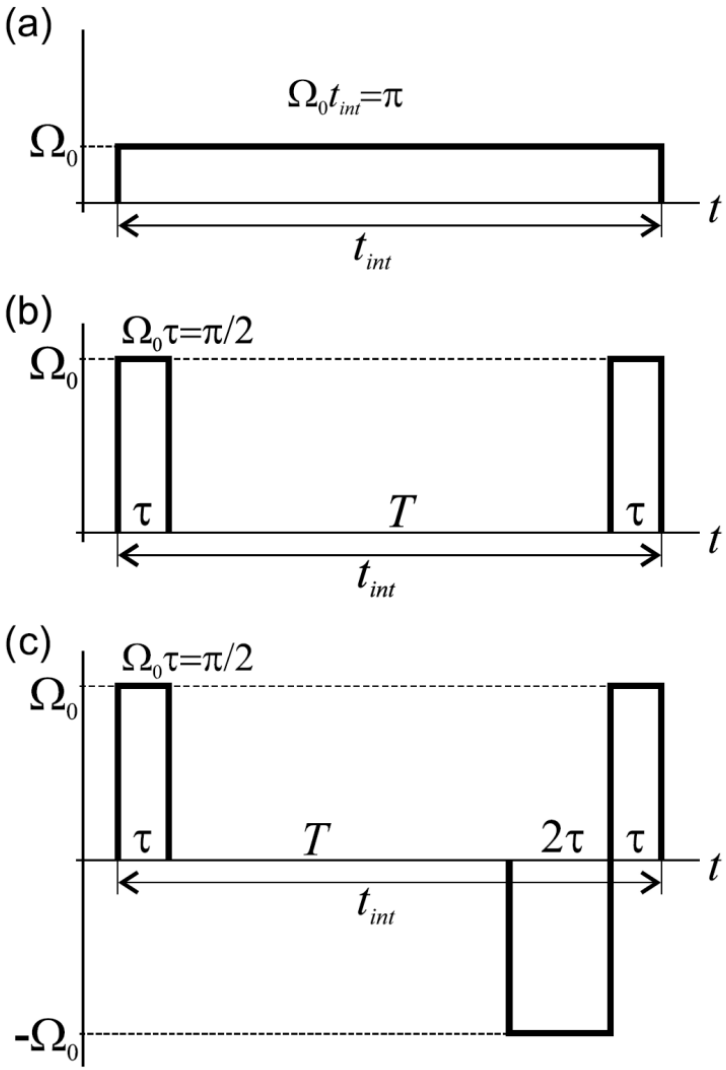
$$\bar{\delta}_{\rm ac}^{(\text{off-res})} = \alpha \frac{|\Omega_0|^2}{|\mathbf{e}\cdot\mathbf{n}_{\rm B}|^2}$$

where α is some proportionality factor, which can be experimentally measured. Therefore, we must always consider the total ac Stark shift

$$\bar{\delta}_{\rm ac}^{(\text{tot})} = \bar{\delta}_{\rm ac}^{(\text{el-ind})} + \bar{\delta}_{\rm ac}^{(\text{off-res})} = |\Omega_0|^2 K$$

$$K = \frac{\Delta_{\rm g} + \Delta_{\rm e}}{4F\Delta_{\rm g}\Delta_{\rm e}}\varepsilon\xi + \alpha$$

Comparative analysis of three different spectroscopic schemes



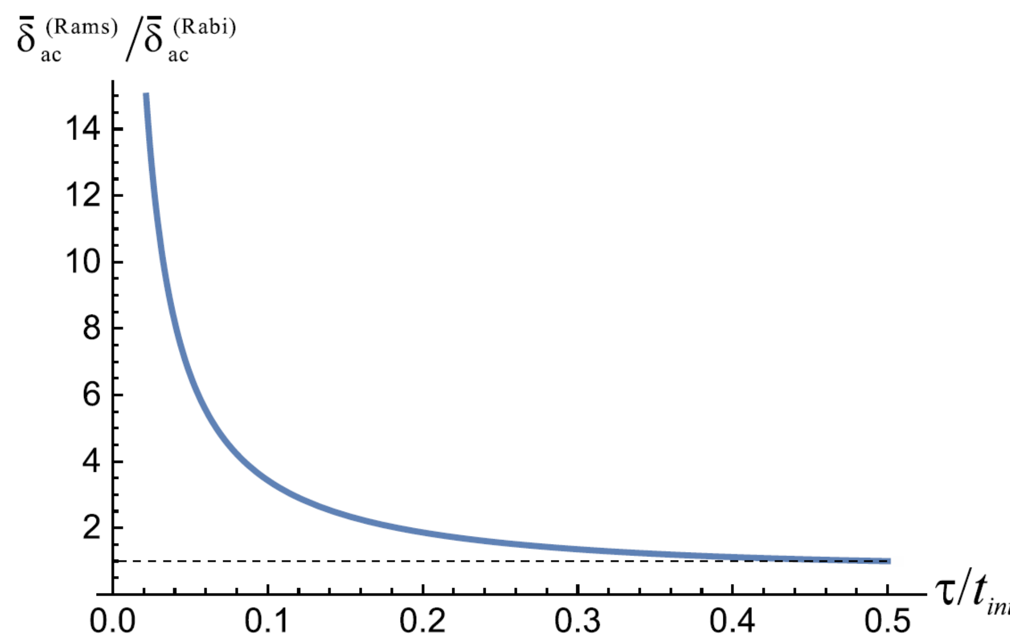
Ac Stark shift for Rabi spectroscopy:

$$\bar{\delta}_{\rm ac}^{\rm (Rabi)} = \bar{\delta}_{\rm ac}^{\rm (tot)} \approx \frac{\pi^2}{t_{int}^2} K$$

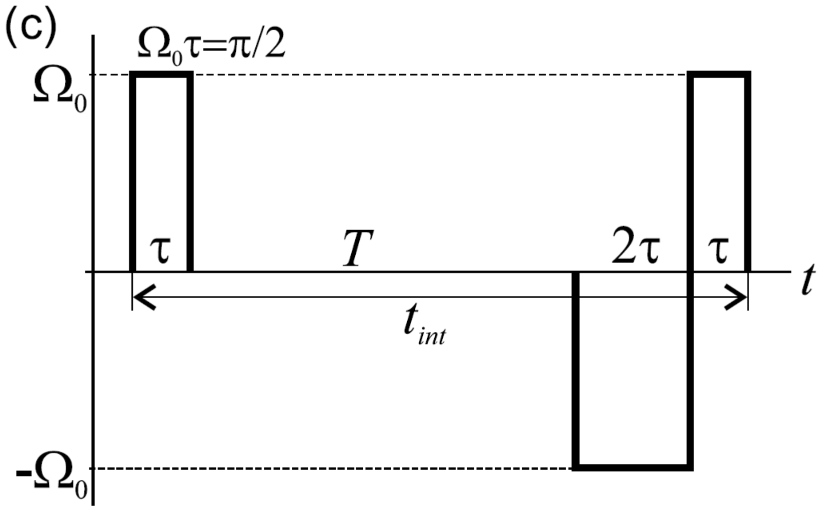
Ac Stark shift for standard Ramsey spectroscopy:

$$\bar{\delta}_{\rm ac}^{\rm (Rams)} \approx \frac{2\bar{\delta}_{\rm ac}^{\rm (tot)}}{2+|\Omega_0|T} = \frac{\pi^2}{2\tau^2[2+\pi T/(2\tau)]} K$$

$$\frac{\bar{\delta}_{\rm ac}^{\rm (Rams)}}{\bar{\delta}_{\rm ac}^{\rm (Rabi)}} = \frac{1}{2(\tau/t_{int})^2\,[2-\pi+0.5\pi(\tau/t_{int})^{-1}]}$$



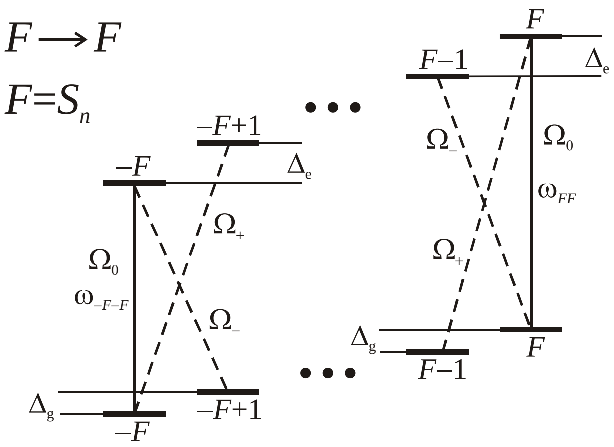
Hyper-Ramsey spectroscopy



$$\bar{\delta}^{(\text{h-Rams})}_{-F-F} \approx \frac{\pi}{T}\left(\frac{\bar{\delta}_{-F-F} + \bar{\delta}^{(\text{off-res})}_{\text{ac}}}{|\Omega_0|}\right)^3,$$

$$\bar{\delta}^{(\text{h-Rams})}_{+F+F} \approx \frac{\pi}{T}\left(\frac{\bar{\delta}_{+F+F} + \bar{\delta}^{(\text{off-res})}_{\text{ac}}}{|\Omega_0|}\right)^3,$$

$$\left|\frac{\bar{\delta}_{\pm F\pm F} + \bar{\delta}^{(\text{off-res})}_{\text{ac}}}{\Omega_0}\right| \sim \left|\frac{\bar{\delta}^{(\text{tot})}_{\text{ac}}}{\Omega_0}\right| < 0.001$$



$$\bar{\delta}^{(\text{h-Rams})}_{\text{ac}} = \frac{\bar{\delta}^{(\text{h-Rams})}_{-F-F} + \bar{\delta}^{(\text{h-Rams})}_{+F+F}}{2}$$

Thus, the ac Stark shift for the hyper-Ramsey scheme

$$\bar{\delta}^{(\text{h-Rams})}_{\text{ac}} = \frac{\bar{\delta}^{(\text{h-Rams})}_{-F-F} + \bar{\delta}^{(\text{h-Rams})}_{+F+F}}{2}$$

becomes significantly lower than 10^{-19} relative to the clock frequency ω_0.

Conclusion

1. We have investigated a previously unconsidered probe field ellipticity-induced ac-Stark shift for atomic lattice-based and ion-trap clocks. It can be called as near resonant ac-Stark shift due to Zeeman structure.
2. This shift arises from the both uncontrolled ellipticity and uncontrolled magnetic field orientation.
3. For the typical experimental conditions with lattice-based clocks this shift can have the relative level of order of 10^{-17}-10^{-19}, i.e. not negligible. Thus, it should be taken into account in the clock budgets.
4. This shift can be carefully investigated experimentally with using of circular polarized probe field ($\sin(2\varepsilon)=1$, to maximize this shift). It can help to find the optimal point of orientation of magnetic field ($\sin(\varphi)=0$).
5. This shift can be suppressed by the use of the hyper-Ramsey method [V.I. Yudin, et.al., PRA **82**, 011804(R) (2010)].

V.I. Yudin, A.V. Taichenachev, O.N. Prudnikov, et al., Probe Field Ellipticity-Induced Shift in an Atomic Clock, Phys. Rev. Appl., accepted for publication (2023).

Thank you very much for your attention!